State-of-the-Art Join Algorithms in Database (Theory) and Lower Bounds from Fine-Grained Complexity

Austen Z. Fan Paraschos Koutris Hangdong Zhao

University of Wisconsin-Madison

DB Affiliates Workshop
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Outline

Vocabularies

Acyclicity takes it all. – Yannakakis Algorithm

How is that different from a tree? – Worst-Case Optimal Join

Always have a plan B! – PANDA

Can we do better? – Lower Bounds from Fine-Grained Complexity

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SELECT t1.A, t1.B, t2.C
FROM Table1 t1
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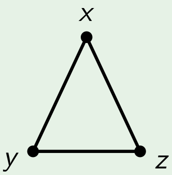

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If H_q is acyclic, then we can evaluate q in linear time $O(|\mathrm{In}| + |\mathrm{Out}|)$.

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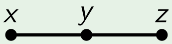

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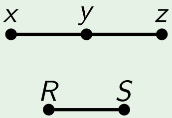

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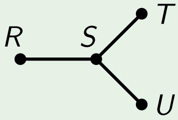

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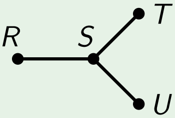

Definition (Join Tree)

A *join tree* for a CQ q is a tree $\mathcal{T}$ whose vertices are the atoms in q such that, for any pair of atoms R, S, all variables common to R and S occur on the unique path connecting R and S.

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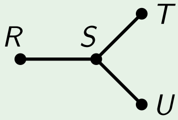

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$q(x, y) :- R(x, y), S(y, z)$, given $|R| = |S| = N$, what's the worst-case output size?

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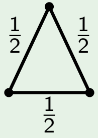

Heavy-Light-Split Query Plan

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Directed 2-paths with intermediate vertices being light: there are $N \cdot \sqrt{N}$ many and they can be found in $O(N \cdot \sqrt{N}) = O(N^{\frac{3}{2}})$ time. For each such path, check whether the endpoints are connected.

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Tree Decomposition

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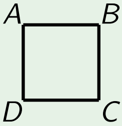

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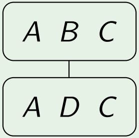

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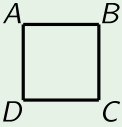
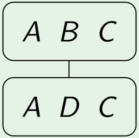
A tree decomposition for 

is 

Tree decomposition = Query plan

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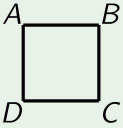
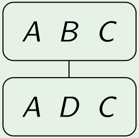
A tree decomposition for 

is 

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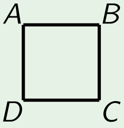
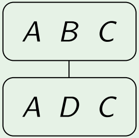
A tree decomposition for 

is 

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Just ensemble tables into bags, run WCOJ on each bag and then run Yannakakis on those bags!

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Two is better than one

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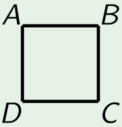
The 4-cycle query



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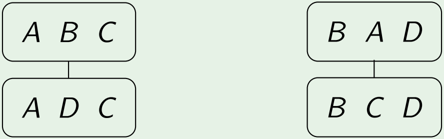

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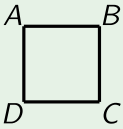

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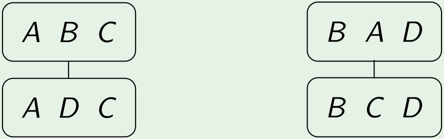

Which one to use?

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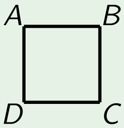

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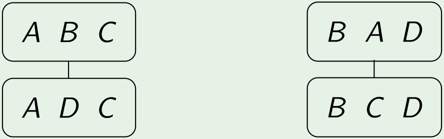

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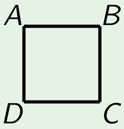

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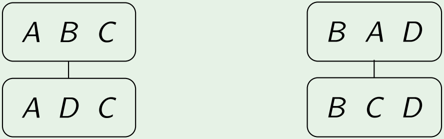

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Use two decompositions at the same time!

PANDA

Example (4-Cycle)

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Using one tree decomposition $\rightarrow O(N^2)$.

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Lemma (Marx, 10')

For any hypergraph $\mathcal{H}$, $\mathrm{subw}(\mathcal{H}) \leq \mathrm{fhtw}(\mathcal{H})$.

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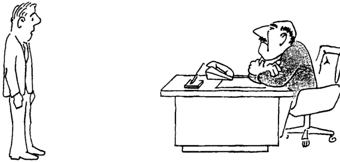

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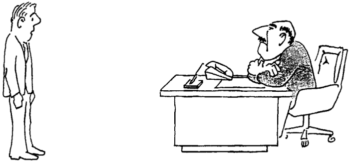

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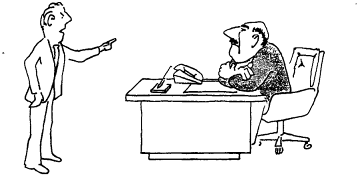

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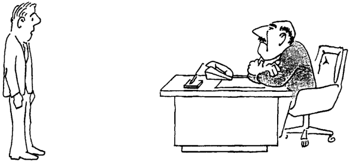

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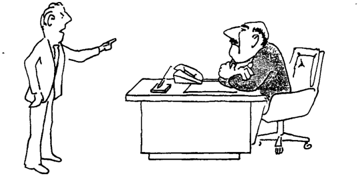
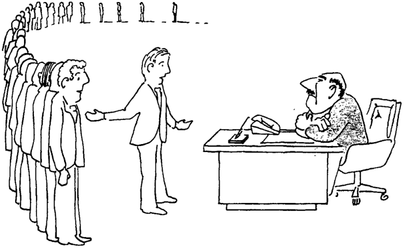


"I can't find an efficient algorithm, because no such algorithm is possible!"



"I can't find an efficient algorithm, but neither can all these famous people."

Lower Bounds

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Theorem (Fan, Koutris & Zhao, 23')

Any q cannot be computed via a combinatorial algorithm in time $O(|I|^{\mathsf{emb}(\mathcal{H}_q)-\epsilon})$ *unless the Combinatorial k-Clique Conjecture is false.*

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	emb	subw
Acyclic	1	1
Chordal	$=$	$=$
ℓ-cycle	$2-1/\lceil \ell/2 \rceil$	$2-1/\lceil \ell/2 \rceil$
$K_{2,\ell}$	$2-1/\ell$	$2-1/\ell$
$K_{3,3}$	2	2
A_ℓ	$(\ell-1)/2$	$(\ell-1)/2$
$\mathcal{H}_{\ell,k}$	ℓ/k	ℓ/k
Q_b	17/9	2
Q_{hb}	7/4	2

Table: Embedding power and submodular width for some query classes

Summary

Vocabularies

Acyclicity takes it all. – Yannakakis Algorithm

How is that different from a tree? – Worst-Case Optimal Join

Always have a plan B! – PANDA

Can we do better? – Lower Bounds from Fine-Grained Complexity

Thank You!

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