Lower Bounds for Sum-Product Queries

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Preliminary Exam

Introduction

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2. Fixing the induced hypergraph → Class of queries

   Bounded arity [Gro07, Mar10] & Unbounded arity [Mar13]

What is the exact lower bound for a given Sum-Product Query?

We partially answer the above question:

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2. Unconditional lower bound via monotone circuits

Outline

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A *Conjunctive Query* Q is an expression associated to a hypergraph $\mathcal{H} = ([n], \mathcal{E})$ where $[n] = \{1, \ldots, n\}$ and some $U \subseteq [n]$:

$$Q(\mathbf{x}_U) \leftarrow \bigwedge_{e \in \mathcal{E}} R_e(\mathbf{x}_e)$$

where each R_e is a relation of arity $|e|$, the variables $x_1, x_2, \ldots, x_n$ take values in some discrete domain, and $\mathbf{x}_e := (x_i)_{i \in e}$.

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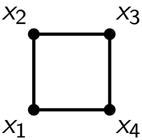

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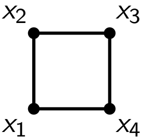

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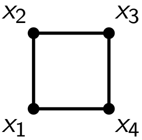

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We are implicitly considering CQ without self-join. We will come back to this point for further work.

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*Fixing relations (*NP*) v.s. fixing hypergraphs (*P*).*

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$\mathbb{T} \leftrightarrow$ optimization

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Compute $\displaystyle\sum_{\substack{V' \subseteq V \\ |V'|=k}} \prod_{\{v,w\} \in V'} \text{weight}(\{v, w\}) \leftrightarrow$ Counting k-clique

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The *provenance polynomial* for a full CQ Q is parameterized by an underlying semiring $\mathbb{S}$, a hypergraph $\mathcal{H}$, and an instance I:

$$p_I^Q := \bigoplus_{t \in Q(I)} \bigotimes_{e \in \mathcal{E}} x^e_{t[e]}$$

where $x^e_{t[e]}$ is a variable that captures the value of the tuple $t[e] \in R_e$ in the semiring domain $\mathbf{D}$ [GKT07].

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When we work over the counting semiring, the provenance polynomial becomes a polynomial:

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$$\begin{aligned} p_I^Q = & (x_{a_1,a_2} \otimes x_{a_2,a_3} \otimes x_{a_3,a_4} \otimes x_{a_4,a_1}) \oplus \\ & (x_{a_1.a_2} \otimes x_{a_2,a_3} \otimes x_{a_3,b_4} \otimes x_{b_4,a_1}) \oplus \\ & (x_{c_1,c_2} \otimes x_{c_2,d_3} \otimes x_{d_3,c_4} \otimes x_{c_4,c_1}) \end{aligned}$$

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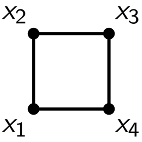
Two tree decompositions for 

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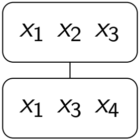

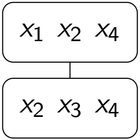

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Remark

The famous Worst-Case Optimal Join achieves $O(m^{\mathrm{fhw}(\mathcal{H})})$ running time for computing $\mathcal{H}$ [NPRR18].

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Remark

It remains open whether computing $\mathrm{entw}(\mathcal{H})$ *is even decidable.*

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A constraint of the form $(X, Y, 1)$ is a Functional Dependency.

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The degree constraints on an instance can be translated as constraints on entropic functions as follows:

$$\mathrm{HDC} := \left\{ h : 2^{[n]} \to \mathbb{R}_+ \mid \bigwedge_{(X,Y,N_{Y|X}) \in \mathrm{DC}} h(Y|X) \leq \log N_{Y|X} \right\}$$

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Hypothesis (Min Weight k-Clique; Lincoln, Vassilevska-Williams & Williams, 17')

Any randomized algorithm to find a k-clique of minimum total edge weight requires $n^{k-o(1)}$ time on a Word RAM model.

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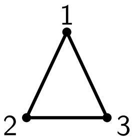

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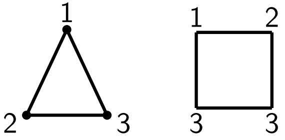

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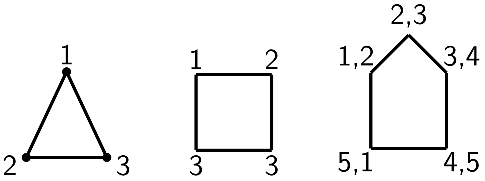

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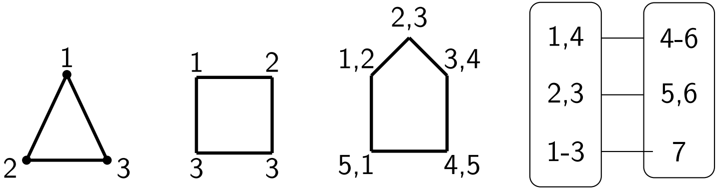

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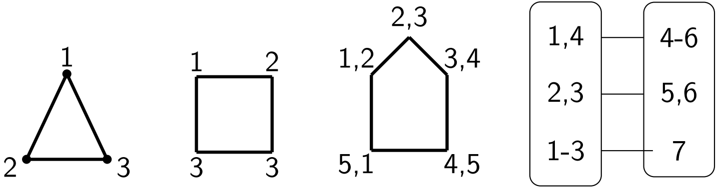

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Theorem (F., Koutris & Zhao, 23')

For any $\mathcal{H}$, CSP($\mathcal{H}$) *cannot be computed via a combinatorial algorithm in time $O(|I|^{\mathrm{emb}(\mathcal{H})-\epsilon})$ unless the Combinatorial k-Clique Conjecture is false.*

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In a very recent work, Bringmann and Gorbachev showed that emb($\mathcal{H}$) *is tight for all* CSP($\mathcal{H}$) *that admits sub-quadratic algorithm [BG24].*

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In fact, it captures all $\mathcal{H}$ that admits sub-quadratic algorithm: If $\mathrm{CSP}(\mathcal{H})$ admits a sub-quadratic algorithm, then $\mathrm{emb}(\mathcal{H}) < 2$, and in that case there exists an $O(|I|^{\mathrm{emb}(\mathcal{H})})$ algorithm.

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Theorem (F., Koutris & Zhao, 23')

For any $\mathcal{H}$, $\mathsf{CSP}(\mathcal{H})$ over tropical semiring cannot be computed via any randomized algorithm in time $O(|I|^{\mathsf{emb}(\mathcal{H})-\epsilon})$ unless the Min Weight k-Clique Conjecture is false.

Examples

	emb	subw
Acyclic	1	1
Chordal	=	=
ℓ-cycle	$2-1/\lceil \ell/2 \rceil$	$2-1/\lceil \ell/2 \rceil$
$K_{2,\ell}$	$2-1/\ell$	$2-1/\ell$
$K_{3,3}$	2	2
A_ℓ	$(\ell-1)/2$	$(\ell-1)/2$
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Table: Clique embedding power and submodular width for some classes of queries

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Table: Clique embedding power and submodular width for some classes of queries

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Bringmann and Gorbachev showed $\Omega(m^2)$ lower bound for both Q_b and Q_{hb} through MinConv conjecture [BG24].

Circuits over Semirings

Recall our provenance polynomial $p_I^Q = \bigoplus_{t \in Q(I)} \bigotimes_{e \in \mathcal{E}} x_{t[e]}^e$.

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A *circuit* F *over a semiring* $\mathbb{S}$ is a Directed Acyclic Graph (DAG) with input nodes variables in a set S_x containing $x^e_{t[e]}$'s and the constants $\mathbf{0}, \mathbf{1}$. Every other node is labelled by $\oplus$ or $\otimes$ and has fan-in 2; these nodes are called $\oplus$-*gates* and $\otimes$-*gates*, respectively.

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A circuit F is said to *compute* a polynomial p if F and p coincide as functions (interpreted over the semiring $\mathbb{S}$), and is said to *produce* a polynomial p if F and p have exactly the same terms, i.e. monomials with their coefficients, syntactically.

Example

$$p_I^Q = (x_{a_1,a_2} \otimes x_{a_2,a_3} \otimes x_{a_3,a_4} \otimes x_{a_4,a_1}) \oplus$$
$$(x_{a_1.a_2} \otimes x_{a_2,a_3} \otimes x_{a_3,b_4} \otimes x_{b_4,a_1}) \oplus$$
$$(x_{c_1,c_2} \otimes x_{c_2,d_3} \otimes x_{d_3,c_4} \otimes x_{c_4,c_1})$$

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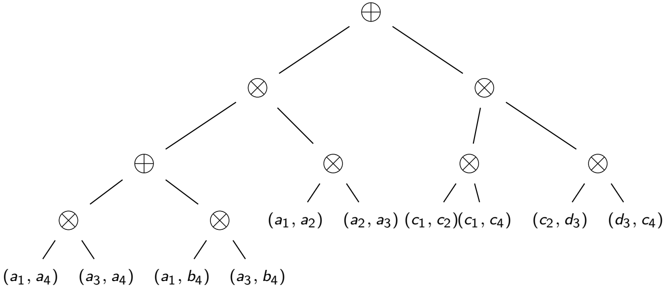
$$\boxed{Q(x_1, x_2, x_3, x_4) \leftarrow R(x_1, x_2), S(x_2, x_3), T(x_3, x_4), U(x_4, x_1)}$$



Motivation

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2. Circuits that compute the provenance polynomial of a CQ can be viewed as a concise representation of the corresponding provenance polynomial interpreted over the given semiring [OZ15, GKT07].

Main Results

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Theorem (F., Koutris & Zhao, 24')

For any $\epsilon > 0$ *and any hypergraph* $\mathcal{H}$*, there exists an instance* I *and* $k > 0$ *that satisfies the constraints* $\mathsf{HDC} \times k$ *such that any circuit* F *that computes the polynomial* $p_I^{\mathcal{H}}$ *over* $\{\mathbb{B}_{\mathsf{lin}}, \mathbb{T}, \mathbb{C}\}$ *has size*

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Theorem (F., Koutris & Zhao, 24')

Let I be any instance that satisfies the degree constraint DC. There exists a multilinear and homogeneous circuit F of size $O(2^{\textit{da-entw}(\mathcal{H}, \mathsf{HDC})})$ that produces the polynomial $p_I^{\mathcal{H}}$ over any idempotent semiring.

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Remark

This notion has been extensively used to prove circuit lower bound [JS82, AI03].

TD from a Parse Tree

For a monomial q in $p_I^{\mathcal{H}}$, we define a structure $\mathcal{T}_q = (\mathcal{T}, \chi)$ inductively in a bottom-up fashion from its parse tree:

1. For an input gate g of the variable $x^e_{t[e]}$, add a node v_g in $\mathcal{T}$ with $\chi(v_g) = e$. The input gate g is said to be *associated to* the node v_g.
2. For a $\oplus$-gate g, associate g to the node that is associated to g's single child in the parse tree.
3. For a $\otimes$-gate g, let g_1 and g_2 be its children. Let q_1, q_2 be the monomials computed at g_1, g_2 respectively; and B_g be the set of vertices $v \in \mathcal{V}(\mathcal{H})$ such that all hyperedges incident to v appear either exclusively in q_1 *or* exclusively in q_2. We add a node v_g with $\chi(v_g) = (\chi(v_{g_1}) \cup \chi(v_{g_2})) \setminus B_g$ as the parent of v_{g_1} and v_{g_2} in $\mathcal{T}$. We associate g with the new node v_g.

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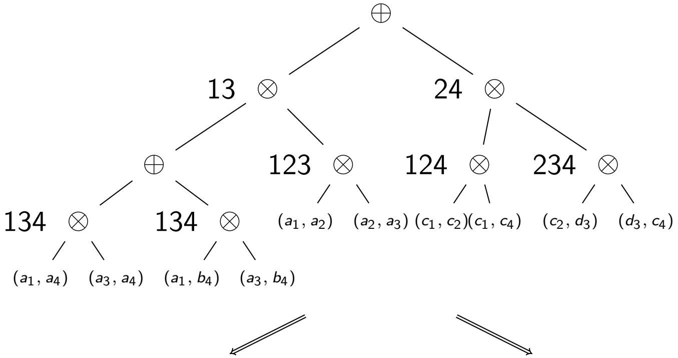
Remark

It is thus possible to assign a type $\mathrm{tp}(g)$ to each $\otimes$-gate g as $\chi(v_g)$ for some decomposition $\mathcal{T}_q = (\mathcal{T}, \chi)$ of a monomial q. In other words, the circuit F yields a globally consistent type assignment to each $\otimes$-gate in F.

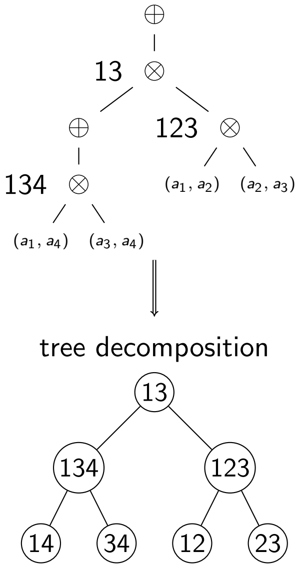
Example

$$Q(x_1, x_2, x_3, x_4) \leftarrow R(x_1, x_2), S(x_2, x_3), T(x_3, x_4), U(x_4, x_1)$$

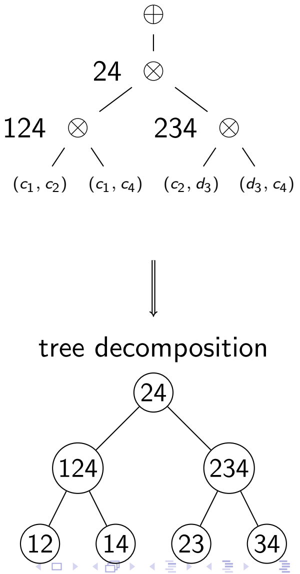


Example

parse tree for (a_1, a_2, a_3, a_4)



parse tree for (c_1, c_2, d_3, c_4)



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- ✗ There exists classes of graphs (e.g. expanders) where the gaps between that variant of clique embedding power and the tree widths are at least quadratic [GM09].
- ? What is the gap between the clique embedding power and submodular width [Mar13]?

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Circuit for CQ with self-joins

- **?** Can we provide tight circuit lower bounds for CQ with self-joins?
- ✓ Interesting connection to the notion of “minimal” queries [CS23] and the characterization of query containment parametrized by the underlying semiring [KRS12].

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? We are investigating the generalization of Bellman-Ford to arbitrary linear Datalog programs to construct logarithmic-depth circuit.

Thank You!

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Theorem (Max, 13')

Let $\mathcal{C}$ be a recursively enumerable class of hypergraphs. Assuming the Exponential Time Hypothesis, $\mathsf{CSP}(\mathcal{C})$ *parametrized by $\mathcal{H}$ is fixed-parameter tractable if and only if $\mathcal{C}$ has bounded submodular width.*

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Informally, ETH says that 3-SAT cannot be solved in $2^{o(n)}$ time and SETH says that k-SAT needs 2^n for large k (when $k \to \infty$).

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