The Fine-Grained Complexity of Boolean Conjunctive Queries and Sum-Product Problems

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ICALP 2023

Outline

Boolean Conjunctive Queries

Sum-of-Product Computation

Fine-Grained Complexity

Our Work

Boolean Conjunctive Queries
- Preliminaries
- Algorithms

Sum-of-Product Computation
- BCQ as CSP
- Semiring framework

Fine-Grained Complexity

Our Work
- Clique embedding power
- Main results
- Tightness and gaps

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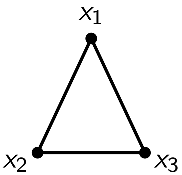

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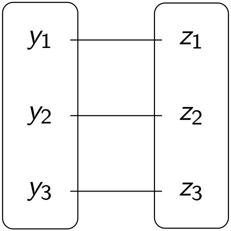

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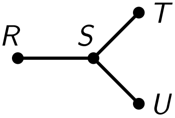

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A *fractional edge cover* of a hypergraph $\mathcal{H} = (\mathcal{V}, \mathcal{E})$ is an assignment from each hyperedge $e \in \mathcal{E}$ to a weight $u_e \in \mathbb{R}_{\geq 0}$, such that for any vertex $v \in \mathcal{V}$ we have $\sum_{e \in \mathcal{E} : v \in e} u_e \geq 1$.

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Theorem (AGM Bound)

Let q be a full CQ with associated $\mathcal{H}$. For every fractional edge cover of $\mathcal{H}$, the output size of q is bounded by $\prod_{e \in \mathcal{E}} N_e^{u_e}$.

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The minimum fractional edge cover number ρ^* of Δ is $\frac{3}{2}$.

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Otherwise, all vertices are heavy: but there are at most $\frac{2N}{\sqrt{N}} = O(\sqrt{N})$ many heavy vertices. Construct the $O(\sqrt{N})$-by-$O(\sqrt{N})$ matrix and use matrix multiplication to find in $O((\sqrt{N})^3) = O(N^{\frac{3}{2}})$ time.

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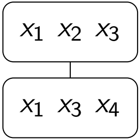
A tree decomposition for  is 

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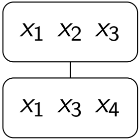
A tree decomposition for  is 

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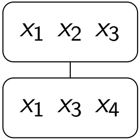
A tree decomposition for  is 

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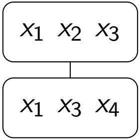
A tree decomposition for  is 

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Definition (Fixed-Parameter Tractable)

Let $\mathcal{C}$ be a class of hypergraphs. CSP($\mathcal{C}$) is said to be *fixed parameter tractable* if there is an algorithm solving every instance I of CSP($\mathcal{H}$) in time $f(\mathcal{H})(||I||)^{O(1)}$, where f is a computable function.

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Theorem (Max, 13')

Let $\mathcal{C}$ be a recursively enumerable class of hypergraphs. Assuming the Exponential Time Hypothesis, CSP($\mathcal{C}$) *parametrized by $\mathcal{H}$ is fixed-parameter tractable if and only if $\mathcal{C}$ has bounded submodular width.*

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Semiring Framework, I

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$$q() :- R_1(\vec{x}_1), R_2(\vec{x}_2), \ldots, R_n(\vec{x}_n)$$

$$q(I) := \bigvee_{v:\text{valuation}} \bigwedge_{i=1}^{n} R_i(v(\vec{x}_i))$$

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$(\{\textsc{True}, \textsc{False}\}, \vee, \wedge) \leftrightarrow$ set semantics

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Example

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$(\mathbb{N}, +, *) \leftrightarrow$ bag semantics
$([0,1], +, *) \leftrightarrow$ probabilistic database

Semiring Framework, II

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Example

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Semiring Framework, II

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Fine-Grained Complexity in 5 Minutes...

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If the edit distance can be solved in time $O(n^{2-\delta})$ for some constant $\delta > 0$, then the Strong Exponential Time Hypothesis is wrong.

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Informally, ETH says that 3-SAT cannot be solved in $2^{o(n)}$ time and SETH says that k-SAT needs 2^n for large k (when $k \to \infty$).

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Conjectures related to k-Clique

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Hypothesis (Min Weight k-Clique; Lincoln, Vassilevska-Williams & Williams, 17')

Any randomized algorithm to find a k-clique of minimum total edge weight requires $n^{k-o(1)}$ time on a Word RAM model.

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Clique Embedding Power, I

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Definition (Touch)

We say $X, Y \subseteq \mathcal{V}$ *touch* in $\mathcal{H}$ if either $X \cap Y \neq \emptyset$ or $\exists e \in \mathcal{E}$ such that $e \cap X \neq \emptyset$ and $e \cap Y \neq \emptyset$.

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Definition (K-Clique Embedding)

A *k-clique embedding* from C_k to $\mathcal{H}$ is a mapping ψ from $v \in [k]$ to a non-empty subset $\psi(v) \subseteq \mathcal{V}$ such that (1) $\forall v, \psi(v)$ induces a connected subhypergraph and (2) $\forall \{v, u\}$, $\psi(v), \psi(u)$ touch in $\mathcal{H}$.

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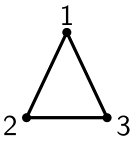

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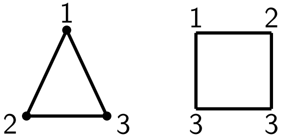

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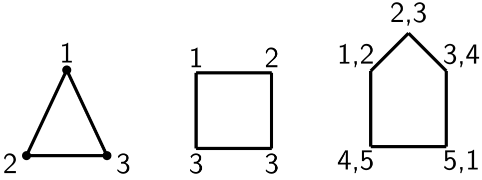

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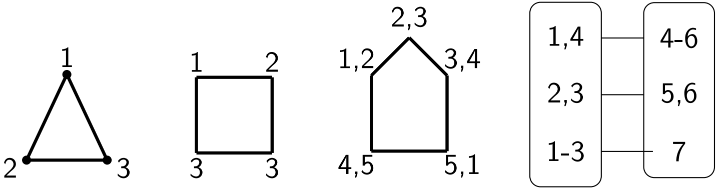

Clique Embedding Power, II

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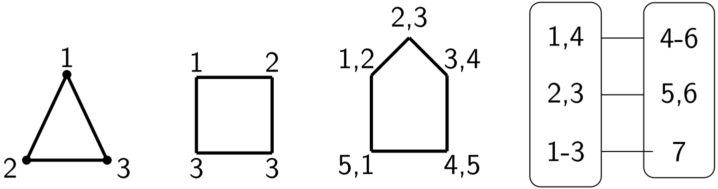

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Theorem (F., Koutris & Zhao, 23')

For any $\mathcal{H}$, CSP($\mathcal{H}$) cannot be computed via a combinatorial algorithm in time $O(|I|^{\mathsf{emb}(\mathcal{H})-\epsilon})$ unless the Combinatorial k-Clique Conjecture is false.

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Proof.

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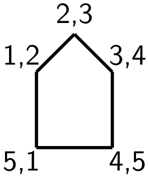

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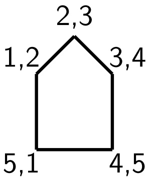

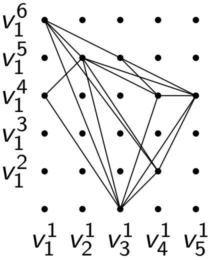

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For any $\mathcal{H}$, $\mathrm{CSP}(\mathcal{H})$ *cannot be computed via a combinatorial algorithm in time* $O(|I|^{\mathrm{emb}(\mathcal{H})-\epsilon})$ *unless the Combinatorial k-Clique Conjecture is false.*

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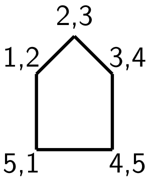

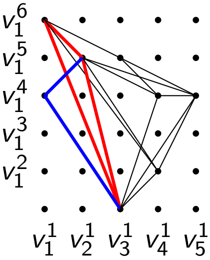

x_1	x_2
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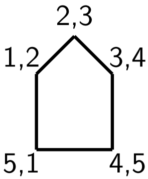

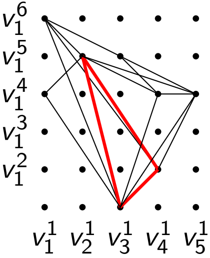

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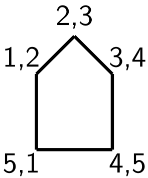

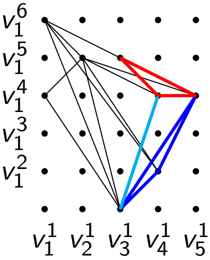

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$\langle v_2^5, v_3^1 \rangle$	$\langle v_3^1, v_4^2 \rangle$

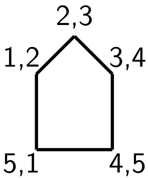
x_3	x_4
$\langle v_3^5, v_4^4 \rangle$	$\langle v_4^4, v_5^4 \rangle$
$\langle v_3^1, v_4^2 \rangle$	$\langle v_4^2, v_5^4 \rangle$
$\langle v_3^1, v_4^4 \rangle$	$\langle v_4^4, v_5^4 \rangle$

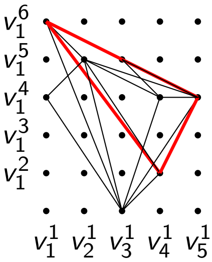
Main Theorem

Theorem (F., Koutris & Zhao, 23')

For any $\mathcal{H}$, CSP($\mathcal{H}$) cannot be computed via a combinatorial algorithm in time $O(|I|^{\mathrm{emb}(\mathcal{H})-\epsilon})$ unless the Combinatorial k-Clique Conjecture is false.

Proof.





x_1	x_2
$\langle v_1^6, v_2^5 \rangle$	$\langle v_2^5, v_3^1 \rangle$
$\langle v_1^4, v_2^5 \rangle$	$\langle v_2^5, v_3^1 \rangle$

x_2	x_3
$\langle v_2^5, v_3^1 \rangle$	$\langle v_3^1, v_4^2 \rangle$

x_3	x_4
$\langle v_3^5, v_4^4 \rangle$	$\langle v_4^4, v_5^4 \rangle$
$\langle v_3^1, v_4^2 \rangle$	$\langle v_4^2, v_5^4 \rangle$
$\langle v_3^1, v_4^4 \rangle$	$\langle v_4^4, v_5^4 \rangle$

x_4	x_5
$\langle v_4^2, v_5^4 \rangle$	$\langle v_5^4, v_1^6 \rangle$

□

Main Theorem

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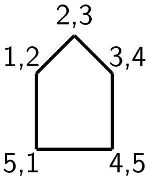
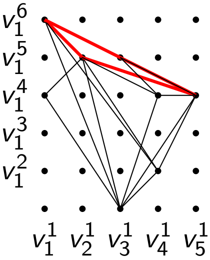




x_1	x_2
$\langle v_1^6, v_2^5\rangle$	$\langle v_2^5, v_3^1\rangle$
$\langle v_1^4, v_2^5\rangle$	$\langle v_2^5, v_3^1\rangle$

x_2	x_3
$\langle v_2^5, v_3^1\rangle$	$\langle v_3^1, v_4^2\rangle$

x_3	x_4
$\langle v_3^5, v_4^4\rangle$	$\langle v_4^4, v_5^4\rangle$
$\langle v_3^1, v_4^2\rangle$	$\langle v_4^2, v_5^4\rangle$
$\langle v_3^1, v_4^4\rangle$	$\langle v_4^4, v_5^4\rangle$

x_4	x_5
$\langle v_4^2, v_5^4\rangle$	$\langle v_5^4, v_1^6\rangle$

x_5	x_1
$\langle v_5^4, v_1^6\rangle$	$\langle v_1^6, v_2^5\rangle$

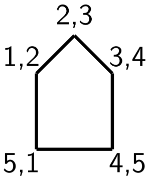
$\square$

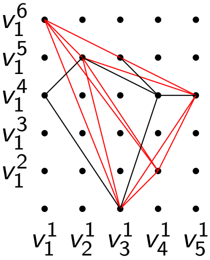
Main Theorem

Theorem (F., Koutris & Zhao, 23')

For any $\mathcal{H}$, CSP($\mathcal{H}$) cannot be computed via a combinatorial algorithm in time $O(|I|^{\text{emb}(\mathcal{H})-\epsilon})$ unless the Combinatorial k-Clique Conjecture is false.

Proof.





x_1	x_2
$\langle v_1^6, v_2^5\rangle$	$\langle v_2^5, v_3^1\rangle$
$\langle v_1^4, v_2^5\rangle$	$\langle v_2^5, v_3^1\rangle$

x_2	x_3
$\langle v_2^5, v_3^1\rangle$	$\langle v_3^1, v_4^2\rangle$

x_3	x_4
$\langle v_3^5, v_4^4\rangle$	$\langle v_4^4, v_5^4\rangle$
$\langle v_3^1, v_4^2\rangle$	$\langle v_4^2, v_5^4\rangle$
$\langle v_3^1, v_4^4\rangle$	$\langle v_4^4, v_5^4\rangle$

x_4	x_5
$\langle v_4^2, v_5^4\rangle$	$\langle v_5^4, v_1^6\rangle$

x_5	x_1
$\langle v_5^4, v_1^6\rangle$	$\langle v_1^6, v_2^5\rangle$

□

Semiring Oblivious Reduction

Semiring Oblivious Reduction

The proof can be adapted to tropical semiring (min k-clique) by assigning each pair $\{u, v\} \subseteq [k]$ to a unique hyperedge according to ψ.

Semiring Oblivious Reduction

The proof can be adapted to tropical semiring (min k-clique) by assigning each pair $\{u, v\} \subseteq [k]$ to a unique hyperedge according to ψ.

Theorem (F., Koutris & Zhao, 23')

For any $\mathcal{H}$, $\mathsf{CSP}(\mathcal{H})$ over tropical semiring cannot be computed via any randomized algorithm in time $O(|I|^{\mathsf{emb}(\mathcal{H})-\epsilon})$ unless the Min Weight k-Clique Conjecture is false.

Boolean Conjunctive Queries
Preliminaries
Algorithms

Sum-of-Product Computation
BCQ as CSP
Semiring framework

Fine-Grained Complexity

Our Work
Clique embedding power
Main results
Tightness and gaps

Summary

	emb	subw
Acyclic	1	1
Chordal	$=$	$=$
ℓ-cycle	$2 - 1/\lceil \ell/2 \rceil$	$2 - 1/\lceil \ell/2 \rceil$
$K_{2,\ell}$	$2 - 1/\ell$	$2 - 1/\ell$
$K_{3,3}$	2	2
A_ℓ	$(\ell - 1)/2$	$(\ell - 1)/2$
$\mathcal{H}_{\ell,k}$	ℓ/k	ℓ/k
Q_b	$17/9$	2
Q_{hb}	$7/4$	2

Table: Clique embedding power and submodular width for some classes of queries

Boolean Conjunctive Queries
- Preliminaries
- Algorithms

Sum-of-Product Computation
- BCQ as CSP
- Semiring framework

Fine-Grained Complexity

Our Work
- Clique embedding power
- Main results
- Tightness and gaps

Thank You!

References I

- Albert Atserias, Martin Grohe, and Dániel Marx, *Size bounds and query plans for relational joins*, SIAM J. Comput. **42** (2013), no. 4, 1737–1767.
- Arturs Backurs and Piotr Indyk, *Edit distance cannot be computed in strongly subquadratic time (unless SETH is false)*, SIAM J. Comput. **47** (2018), no. 3, 1087–1097.
- Todd J. Green, Gregory Karvounarakis, and Val Tannen, *Provenance semirings*, PODS, ACM, 2007, pp. 31–40.
- Martin Grohe, *The complexity of homomorphism and constraint satisfaction problems seen from the other side*, J. ACM **54** (2007), no. 1, 1:1–1:24.

References II

Mahmoud Abo Khamis, Hung Q. Ngo, and Dan Suciu, *What do shannon-type inequalities, submodular width, and disjunctive datalog have to do with one another?*, PODS, ACM, 2017, pp. 429–444.

Andrea Lincoln, Virginia Vassilevska Williams, and R. Ryan Williams, *Tight hardness for shortest cycles and paths in sparse graphs*, SODA, SIAM, 2018, pp. 1236–1252.

Dániel Marx, *Tractable hypergraph properties for constraint satisfaction and conjunctive queries*, J. ACM **60** (2013), no. 6, 42:1–42:51.

Hung Q. Ngo, Ely Porat, Christopher Ré, and Atri Rudra, *Worst-case optimal join algorithms*, J. ACM **65** (2018), no. 3, 16:1–16:40.

References III

Mihalis Yannakakis, *Algorithms for acyclic database schemes*, VLDB, IEEE Computer Society, 1981, pp. 82–94.